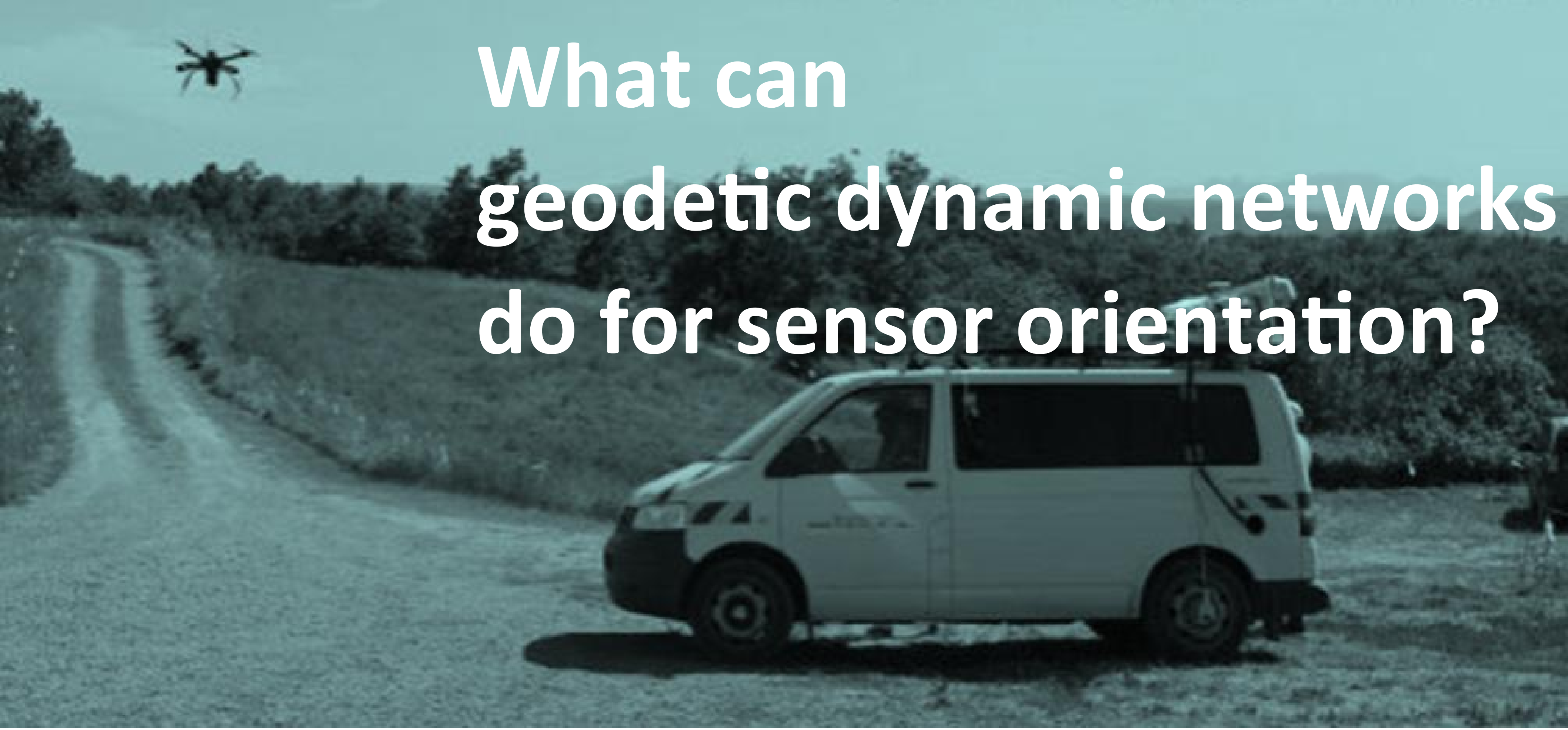
What can geodetic dynamic networks do for sensor orientation?



GeoNumerics
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ISPRS Geospatial Week 2019
EuroCOW 2019
EuroCOW-M3DMaN session
Navigation and Dynamic Networks

Enschede, 2019-06-13

geonumerics

what is this?

$$
\begin{aligned}
\dot{x}^e &= v^e \\
\dot{v}^e &= R_b^e f^b - 2\Omega_{ie}^e v^e + g^e(x^e) \\
\dot{R}_b^e &= R_b^e \left(\Omega_{ei}^b + \Omega_{ib}^b\right)
\end{aligned}
$$

- a 1st-order differential equation
- an observation equation that relates measurements (red) to parameters
- a stochastic differential equation
- the strapdown inertial mechanisation equations
- that can be solved in real-time (navigation) or in post-processing (orientation/ positioning)

difference between positioning and navigation

- navigation is knowing "where" –e.g. *SE(3)*– you are in real-time

  navigation is a polysemous word (e.g. robotic navigation)
  for "us" navigation is not "guidance and control"

- positioning is knowing "where" you are or were

  sensor orientation is positioning in *SE(3)*

- real-time is, usually, demanding in terms of computational speed

- post-mission is, usually, demanding in terms of accuracy

- the estimation/numerical methods of real-time & post-mission may differ

diff... between inertial navigation & inertial thinking

- **inertial navigation** is computing

  $$PVA(t+\Delta t) \text{ from } PVA(t)$$

  using inertial measurements

  $$(\omega,f)(t) \text{ and/or } (\omega,f)(t+\Delta t)$$

- **inertial thinking** is computing

  $$PVA(t+\Delta t) \text{ from } PVA(t)$$

  using the method

  $$M(t-\infty)$$

  because others were doing it in the past

a non-inertial approach to inertial tPVA orientation

$$
\begin{aligned}
\dot{x}^e &= v^e \\
\dot{v}^e &= R_b^e f^b - 2\Omega_{ie}^e v^e + g^e(x^e) \\
\dot{R}_b^e &= R_b^e \left(\Omega_{ei}^b + \Omega_{ib}^b\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{-x^e(t+2\Delta) + 8x(t+\Delta) - 8x(t-\Delta) - x(t-\Delta))}{12 \cdot \Delta} &= v^e \\
\frac{v^e(t+\Delta) - v^e(t-\Delta)}{2 \cdot \Delta} &= R_b^e f^b - 2\Omega_{ie}^e v^e + g^e(x^e) \\
\frac{R_b^e(t-2\Delta) - 4R_b^e(t-\Delta) + 3R_b^e(t)}{2 \cdot \Delta} &= R_b^e \left(\Omega_{ei}^b + \Omega_{ib}^b\right)
\end{aligned}
$$

a non-inertial approach to inertial tPVA orientation

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\begin{aligned}
\dot{x}^e &= v^e \\
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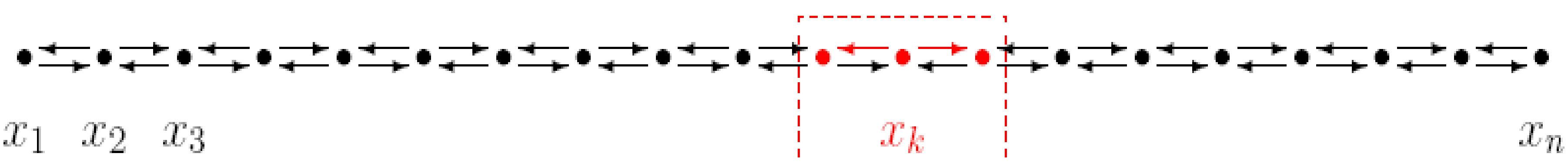
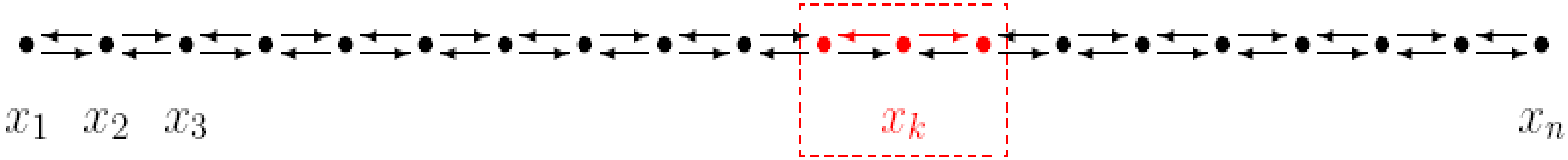
\dot{R}_b^e &= R_b^e \left(\Omega_{ei}^b + \Omega_{ib}^b\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{-x_{k+2}^e + 8x_{k+1}^e - 8x_{k-1}^e + x_{k-2}^e}{12 \cdot \Delta} &= v_k^e \\
\frac{v_{k+1}^e - v_{k-1}^e}{2 \cdot \Delta} &= R_{b\,k}^e f^b - 2\Omega_{ie}^e v_k^e + g^e(x_k^e) \\
\frac{R_{b\,k-2}^e - 4R_{b\,k-1}^e + 3R_{b\,k}^e}{2 \cdot \Delta} &= R_{b\,k}^e \left(\Omega_{ei}^b + \Omega_{ib}^b\right)
\end{aligned}
$$



a non-inertial approach to inertial tPVA orientation



- time steps shall not be necessarily constant
- finite difference coefficients shall not be necessarily centered
- order of approximation shall not be always de same
- higher-order derivatives can also be approximated by finite differences

dynamic geodetic networks

classical geodetic network

$$f(\ell + v, x) = 0$$

ℓ, v, x : random variables

dynamic geodetic network

$$f(\ell(t) + v(t), x(t), \dot{x}(t)) = 0$$

$\ell(t), v(t), x(t)$: stochastic processes

Colomina,I., Blázquez,M., 2004. A unified approach to static and dynamic modelling in photogrammetry and remote sensing. International Archives of Photogrammetry, Remote Sensing and Spatial Information Sciences, Vol. 35-B1, Comm. I, pp. 178-183.

Térmens,M.A., Colomina,I, 2004. Network approach versus state-space approach for strapdown inertial kinematic gravimetry. Gravity, Geoid and Space Missions - GGSM2004, 2004-08-30 / 2004-09-03, Porto.

Colomina,I., Blázquez,M., 2005. On the stochastic modeling and solution of time dependent networks. VI International Geomatic Week, 2005.2.8{11, Barcelona.

dynamic geodetic networks: original motivation

- improvement of mobile mapping trajectories in urban environments
- integration of inertial and photogrammetric measurements
- global –as opposed to stripwise– airborne SINS gravimetric methods

Térmens,M.A., 2014. A network approach for strapdown inertial kinematic gravimetry, PhD Dissertation, PhD Programme Applied Mathematics, Universitat Politècnica de Catalunya, Dep. de Matemàtica Aplicada IV, pp. 378. http://www.tdx.cat/handle/10803/145319.

advantages of dynamic geodetic networks

- **stochastic constraints between parameters (states) at different times**
  crossovers; rigorous, global "loop closures"

- **integration at the raw measurements' level**
  better modelling, correlations are not neglected
  easier outlier detection
  software concentration

- **other modelling advantages**
  the random constant vs. random walk dilemma level

- **availability of geodetic network techniques and tools**

- **related well known numerical methods from ODEs, PDEs and civil engineering**

challenges of dynamic geodetic networks

- **size of matrices / number of measurements and unknowns**
  2 h * 3600 s/h * 200 tωf / s * 15+ parameters/tωf = 21.600.000 + unknowns
  can be reduced (10-100 factor), depending on motion frequency by "downsampling"

$$\Omega\left(\frac{\Delta t}{2}\right) = \frac{\log\left(R(0)^T R(\Delta t)\right)}{\Delta t} \qquad \log R = \frac{\theta}{2\sin\theta}(R - R^T)$$

- **sorting of unknowns**
  new strategies
- **Ill conditioning of normal equations**
  next presentation
- **low redundancy / weak geometry of the INS/GNSS problem**

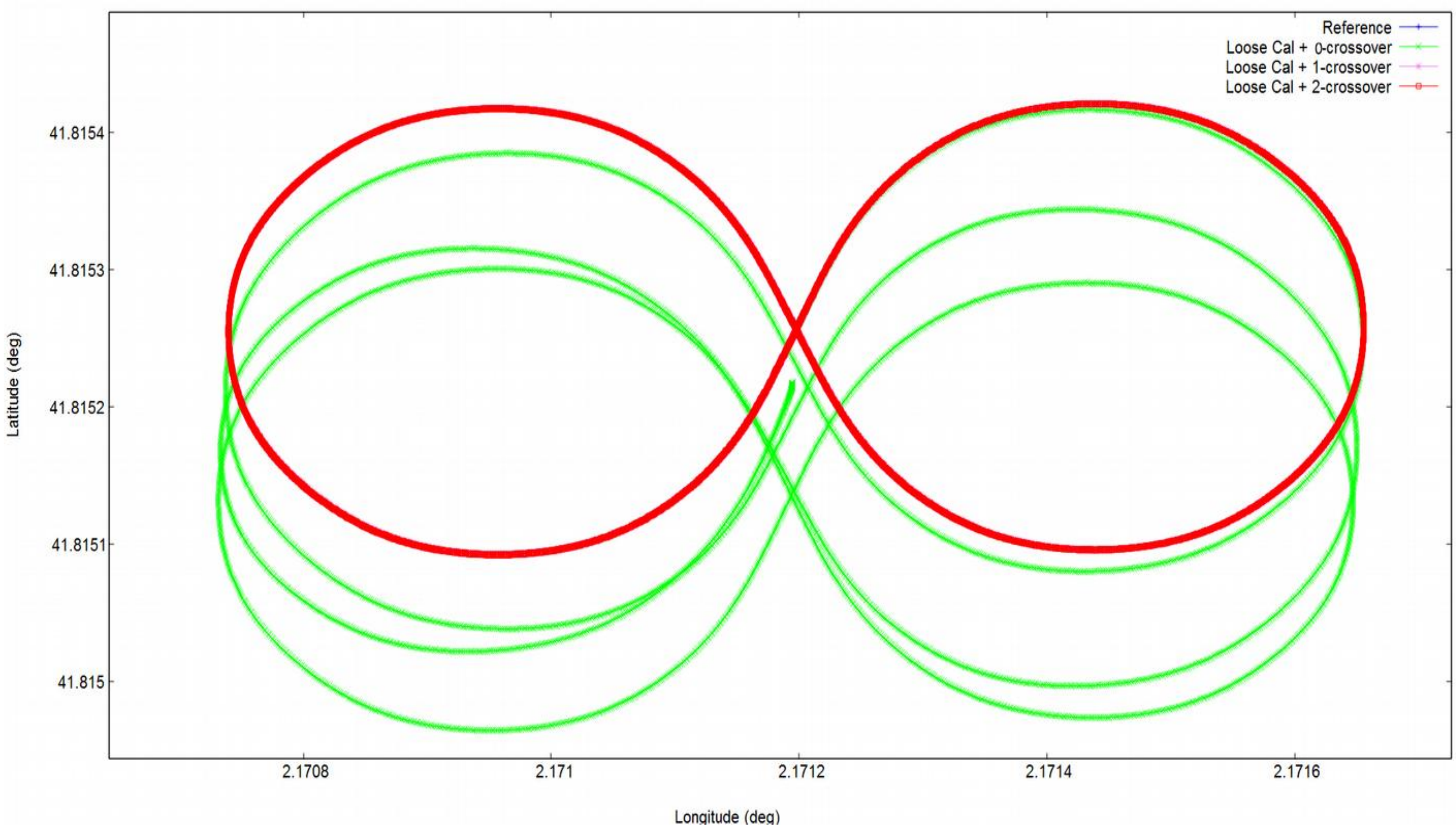
- **the old, known and overlooked wrong Taylor expansions of stochastic processes**

application to terrestrial mobile mapping: crossovers

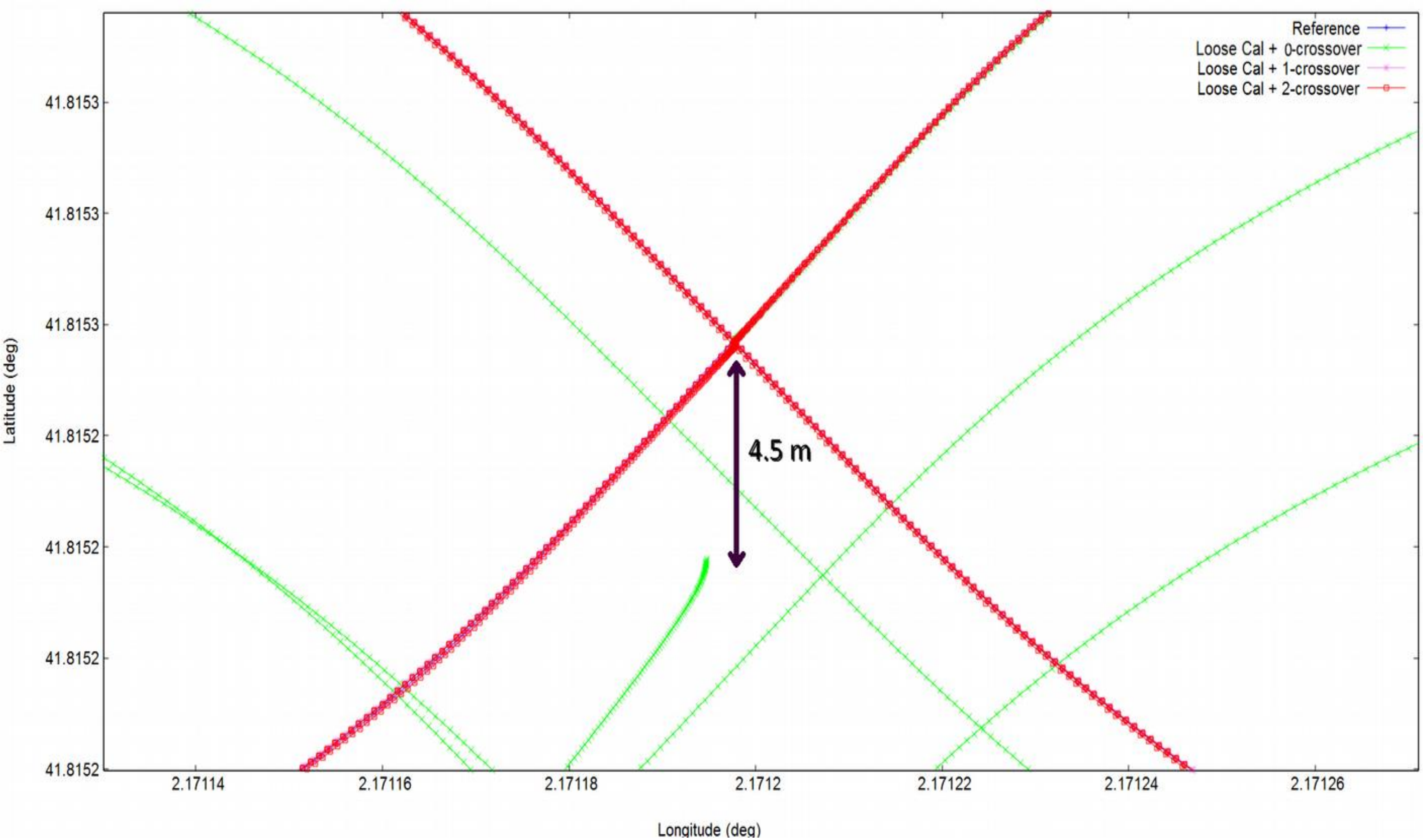
gyro noise	accel noise	gyro repeatability	accel repeatability
0.09 deg / sqrt(h)	0.008 m/s/sqrt(h)	1 deg/s (0.05) 1-σ	4 mg 1-σ

- 3 ∞- shaped loops in *SE(3)* (3D space)
- 50 s length, 20 m radius
- 6 crossovers (only 2 used)
- initial 2 s "calibration"
- coordinate systems: cartesian (x,v) e-frame & exponential (R, *SO(3)*)
- processing: GeoNumerics' IMU simulator, GENA and dynamicSURVEY toolbox
- data validation: GN's NEXA (Predictor-Corrector 4) and dynamicSURVEY toolbox

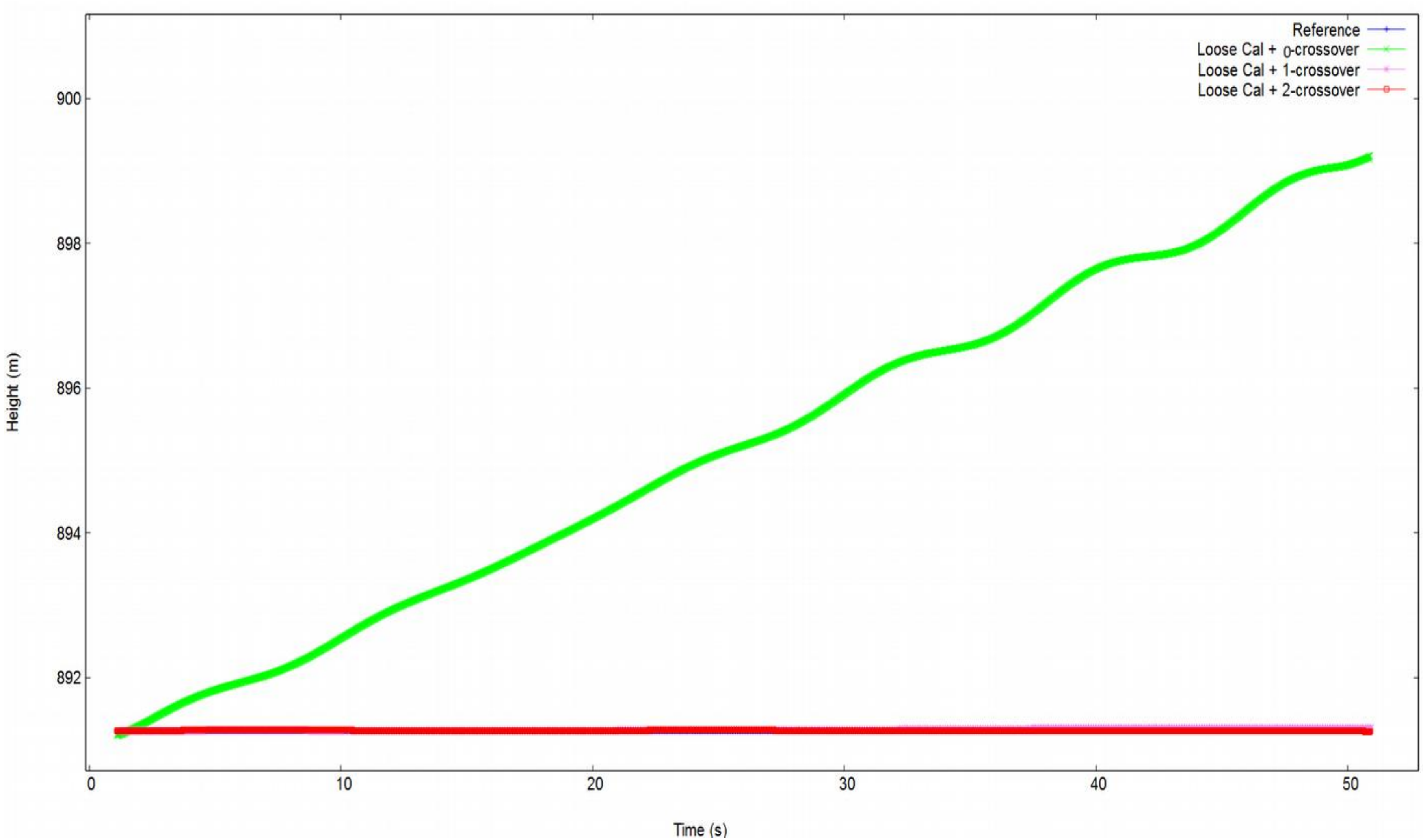
reference, 0, 1 & 2 crossovers



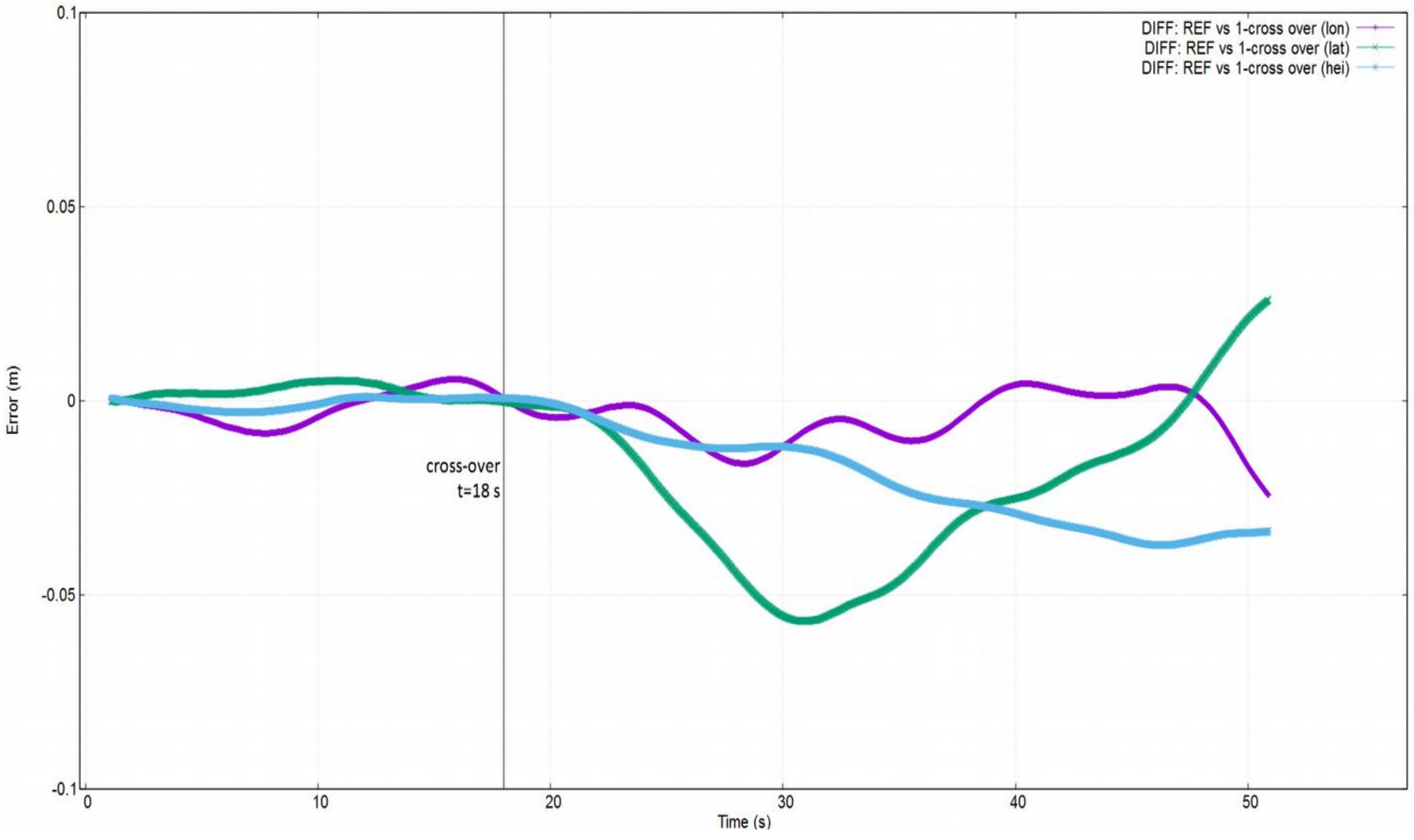
reference, 0, 1 & 2 crossovers



reference, 0, 1 & 2 crossovers: height error

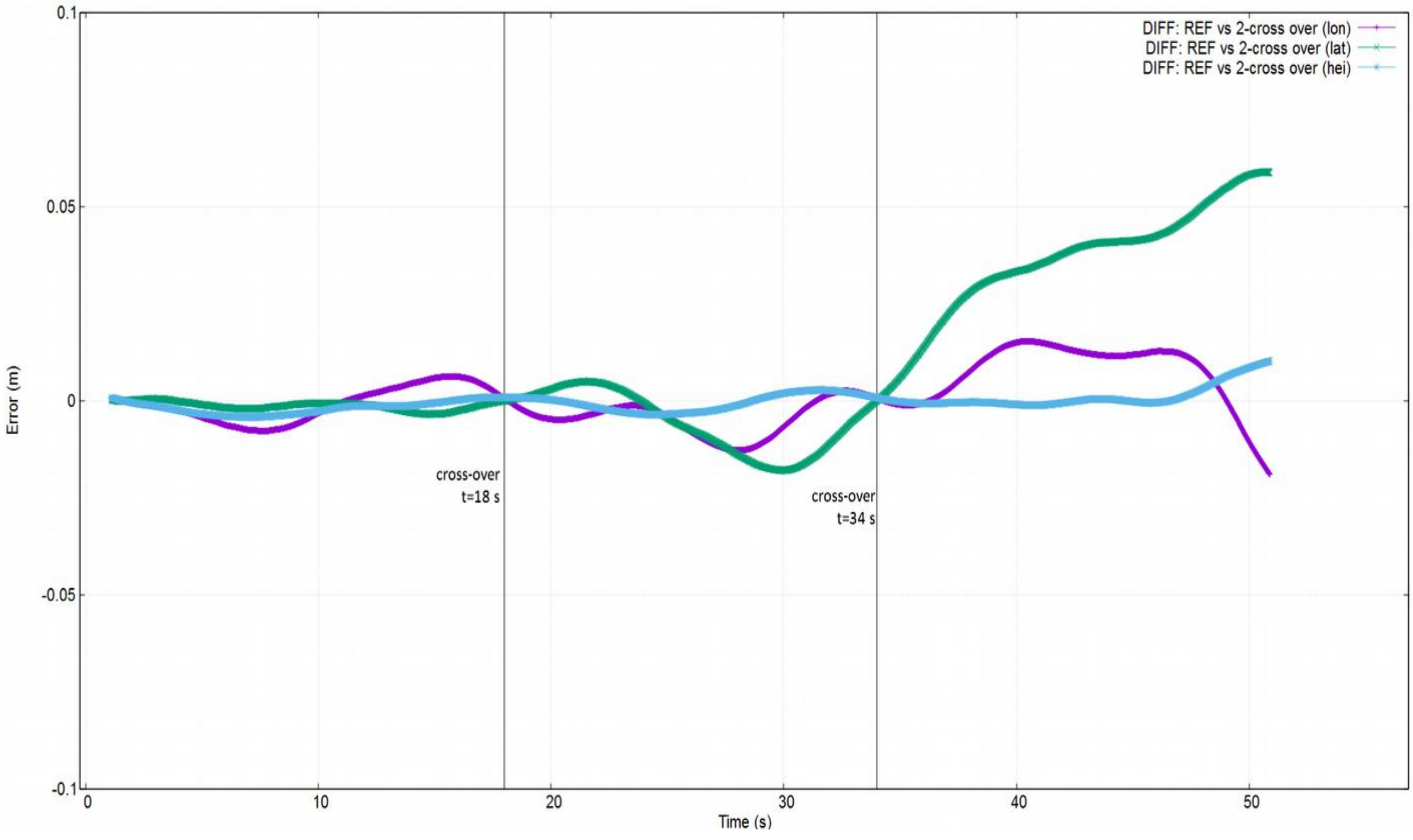


1 crossover



DIFF: REF vs 1-cross over (lon)
DIFF: REF vs 1-cross over (lat)
DIFF: REF vs 1-cross over (hei)
0.1
0.05
0
-0.05
-0.1
Error (m)
cross-over
t=18 s
0
10
20
30
40
50
Time (s)

2 crossovers



application to airborne gravimetry

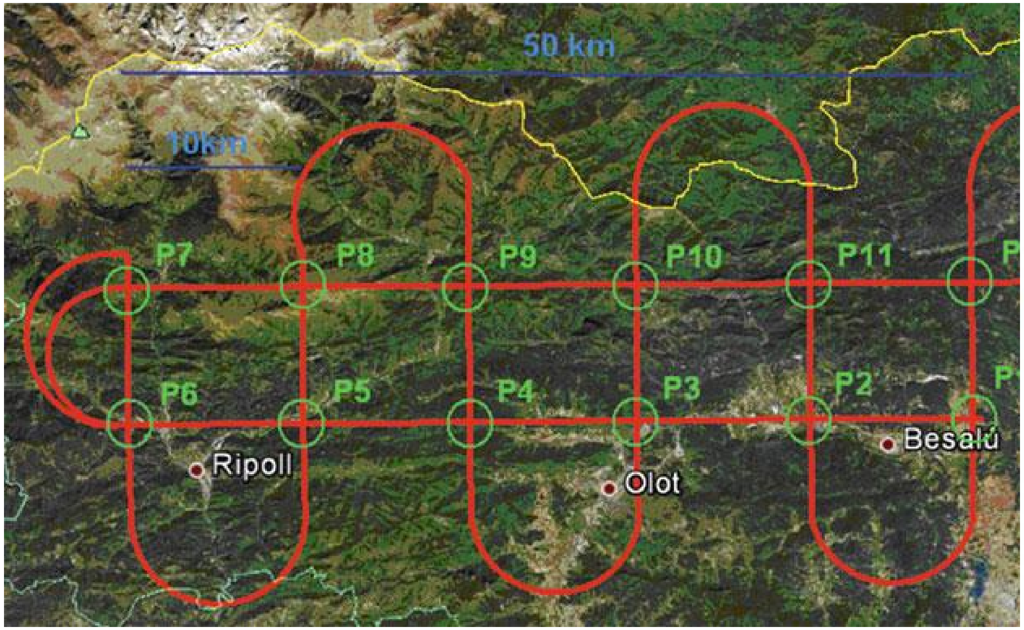
airborne gravimetry: kinematic gravimetry from aircraft.

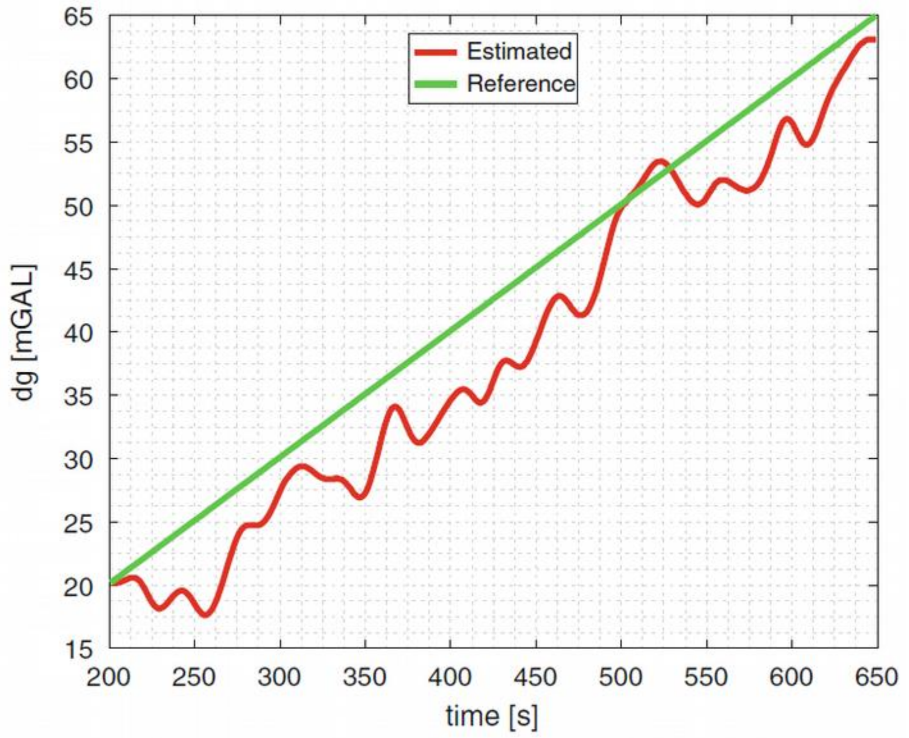
SINS gravimetry: kinematic gravimetry with strapdown IMUs and GNSS.

$$
\begin{aligned}
\ddot{x}^e &= R_b^e f^b - 2\Omega_{ie}^e v^e + g^e(x)^e + \delta g^e(x^e) \\
\dot{R}_b^e &= R_b^e \left(\Omega_{ei}^b + \Omega_{ib}^b\right)
\end{aligned}
$$

- removal of strip gravity-disturbance-vector biases using the flight line crossovers

Galileo for Gravity (GAL) EU-funded project





Skaloud,J., Colomina,I., Parés, M.E., Blázquez,M., Silva,J., Chersich,M., 2016. A method of airborne gravumetry by combining strapdown inertial and new satellite observations via dynamic networks. International Symposium on Earth and Environmental Sciences for Future Generations, International Association of Geodesy Symposia, Springer, Berlin, Heidelberg, pp. 111-122.

application to photogrammetry

Cucci,D., Rehak,M., Skaloud, J., 2017. Bundle adjustment with raw inertial observations in UAV applications. ISPRS Journal of Photogrammetry and Remote Sensing, Vol. 130, pp. 1-12.

Cucci,D., Skaloud,L., 2019. On raw inertial measurements in dynamic networks. EuroCOW 2019.

summary

- reviewed the concept of dynamic networks: clear advantages and some issues
- an old idea seems to go into practice as a geodetic post-processing method for refinement of sequential KF or SLAM results.
- 3 original visions validated/demonstrated:
  - mobile mapping (esp. in GNSS unfriendly environments),
  - Airborne SINS gravimetry and
  - integration with photogrammetric networks.
- more on next presentation...

Thanks!

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